

A COUNTER EXAMPLE IN TOPOLOGY OF STAR-SPACES

PRASENJIT BAL
SUBRATA BHOWMIK*

Department of Mathematics, Tripura University
Suryamaninagar, Tripura, INDIA-799022
balprasenjit177@gmail.com
subrata_bhowmik_math@rediffmail.com*

Abstract

E.K. van Douwen, G.M. Reed, A.W. Roscoe, I.J. Tree in their paper [1] studied so many properties of strongly star-compact(strongly-1-star-compact) spaces and star-Lindelöf(strongly-1-star-Lindelöf) spaces. Y. Song studied many more properties of strongly star-Lindelöf spaces in his papers [2,3,4]. In this paper we will show that there is a topological space which is strongly star-Lindelöf but not strongly star-compact.

keywords: Star-Lindelöf space, star-compact space.

2010 Mathematics subject classification : 54A05, 54B10.

Introduction

For a space X , if $A \subseteq X$ and $\mathcal{U}$ be a collection of subsets of X , then $St(A, \mathcal{U}) = \cup\{(A \cap U) : U \in \mathcal{U}\}$. Strongly star-compact and strongly star-Lindelöf spaces raised from this star operator. In this paper we are to find a space which is strongly star-Lindelöf but not strongly star-compact.

1 Preliminaries

Definition 1.1. A space X is strongly star-compact [1,4] if for every open cover $\mathcal{U}$ of X there exists a finite subset A of X such that $X = St(A, \mathcal{U})$.

Definition 1.2. A space X is strongly star-Lindelöf [1,2,3,4] if for every open cover $\mathcal{U}$ of X there exists a countable subset A of X such that $X = St(A, \mathcal{U})$.

2 A counter example

From the definition of strongly star-compact and strongly star-Lindelöf spaces it is obvious that every strongly star-compact space is strongly star-Lindelöf. Here we are about to show that every strongly star-Lindelöf space may not be strongly star-compact.

Example 2.1. We show that there is a topological space which is strongly star-Lindelöf but not strongly star-compact.

$\Rightarrow$

Let $\mathbb{I} = \{0, 1\}$ $|\mathbb{I}| = 2$, $|\mathbb{I}^\omega| = 2^\omega = \omega_1$.

Clearly, every element of $\mathbb{I}^\omega$ has countable number of components.

Let

$$A_0 = \{\bar{0} = (0, 0, 0, \dots) \in \mathbb{I}^\omega\},$$

$$A_1 = \{(a_1, a_2, a_3, \dots) \in \mathbb{I}^\omega : \text{only one of } a_i = 1\},$$

$$A_2 = \{(a_1, a_2, a_3, \dots) \in \mathbb{I}^\omega : \text{only two of } a_i = 1\},$$

.

.

.

$$A_n = \{(a_1, a_2, a_3, \dots) \in \mathbb{I}^\omega : \text{only } n \text{ number of } a_i\text{'s} = 1\},$$

.

.

.

Let $Y = \{A_1, A_2, A_3, \dots, A_n, \dots\}$ and $Z = P(Y)$. Clearly $|Y| = \omega$ and $|Z| = \omega_1$ (under continuum hypothesis). Let $\mathcal{T} = \{\cup\{A : A \in M\} : M \in Z\}$.

Clearly $\mathcal{T}$ is a topology on $\mathbb{I}^\omega$,. Let $\mathcal{U}$ be an open cover of $\mathbb{I}^\omega$. Then there exists $U_1, U_2, U_3, \dots, U_n, \dots \in \mathcal{U}$ all of which may not be distinct such that $A_i \leq U_i$ for all $i \in \omega$. Let $\mathcal{V} = \{A_i : i \in \omega\}$.

We take the following elements:

$$p_0 = (0, 0, 0, \dots) \in A_0$$

$$p_1 = (1, 0, 0, 0, \dots) \in A_1$$

$$p_2 = (1, 1, 0, 0, 0, \dots) \in A_2$$

.

.

.

$$p_n = (1, 1, 1, \dots, 1(\text{upto } n\text{-terms}), 0, 0, 0, \dots) \in A_n$$

.

.

.

and put $F = \{p_0, p_1, p_2, \dots, p_n, \dots\}$. Thus F is a countable set .

$\therefore St(F, \mathcal{V}) = \cup_{i \in \omega} A_i \subseteq \cup_{i \in \omega} U_i \subseteq \mathbb{I}^\omega$. But $\cup_{i \in \omega} A_i = \mathbb{I}^\omega$

$\Rightarrow St(F, \mathcal{U}) = \cup_{i \in \omega} U_i = \mathbb{I}^\omega$.

$\therefore \mathbb{I}^\omega$ with the topology mentioned above is a strongly star-Lindelöf space.

Now $\mathcal{U}' = \{A_0, A_1, A_2, \dots, A_n, \dots\}$ is also an open cover of $\mathbb{I}^\omega$. For this cover we can not find a finite set E such that $St(E, \mathcal{U}') = \mathbb{I}^\omega$. $\therefore \mathbb{I}^\omega$ with the topology mentioned above is not a strongly star-compact space.

References

1. E.K. van Douwen, G.M. Reed, A.W. Roscoe, I.J. Tree, *Star covering properties*, Topology and its Applications, **39** (1991) 71-103.
2. Yan-Kui Song, *Embedding into discretely absolutely star-Lindelöf spaces II*, Applied General Topology, Universidad Politécnica de Valencia Volume 10, No. **1**, (2009) pp. 13-20.
3. Yan-Kui Song, *A Notes On Discretely Absolutely Star-Lindelöf Spaces*, MATEMATIQKI VESNIK **63**, **3** (2011), 219222.
4. Yan-Kui Song, *Remarks on neighborhood star-Lindelöf spaces II*, Published by Faculty of Sciences and Mathematics, University of Niš, Serbia, (2013), 875880 DOI 10.2298/FIL1305875S.